Proceedings of the 26th International Conference on Offshore Mechanics and Arctic Engineering
OMAE2007
June 10-15, 2007, San Diego, California, USA

OMAE2007-29315

TIME DOMAIN SIMULATION OF THE 3D BENDING HYSTERESIS BEHAVIOUR OF AN UNBONDED FLEXIBLE RISER

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ABSTRACT

This paper presents a 'state of art' in the development of the time domain dynamic simulation of 3D bending hysteresis behaviour of a flexible riser under offshore environment loading. The main technical challenge is to understand and model the riser tensile armour behaviour under continuous changes in both the magnitude and direction of bending, and its subsequent impact on the riser's bending hysteresis characteristics. Because of this technical obstacle, the current industry practice is to model the riser as a linear structure, with certain conservatism enforced, and then to extract the global dynamic loads to a detailed local model for stress and life assessment.

This paper introduces two 3D flexible riser bending hysteresis models, developed by Wellstream and Orcina respectively, and their calibrations against the bending hysteresis loops measured in full scale tests. Both models are implemented using the analysis program OrcaFlex; the Wellstream model is a detailed model that calculates both the total bending moment and the stresses in the tensile armour; the Orcina model is a simpler model that only calculates the total bending moment. A study is presented to illustrate the difference in the riser dynamic responses with and without consideration of the bending hysteresis behaviour, and to assess the difference in the dynamic responses between the Wellstream and Orcina 3D bending hysteresis models.

This development permits more realistic riser structural properties to be modeled in the dynamic simulation, and reports detailed time history stress or strain results of the strength components of the riser. This expands the current practice of riser fatigue analysis of only using the regular wave approach, to using an irregular wave approach employing the rainflow counting method.

INTRODUCTION

The structure of unbonded flexible risers gives a non-linear and hysteretic response when subject to bending. In analysis software this makes it much harder to model such pipes than it is to model steel or titanium pipes. This paper describes two 3D mathematical models of the bending response of unbonded flexibles and compares the results of the models to results from measurements of real Wellstream flexible pipes. Both models have been implemented using the analysis program OrcaFlex.

The first model, developed by Orcina, is a vector hysteresis model built into the time domain simulation program OrcaFlex [1]. The model extends single-plane bending moment response data to bending in 3D. It takes as input a user-specified non-linear bending moment response curve for bending in a single plane. It then applies this data to calculate the bending moment response to fully 3D curvature (i.e. where there may be changes to the plane of curvature). The model does not calculate stresses in the tensile wires, and so cannot be used as the basis for a fatigue analysis of those wires.

The second model, developed by Wellstream, is a detailed model of the way the tensile wires in the structure respond when the pipe is bent. Its outputs include both the 3D bending moment response of the pipe and also the stresses generated in individual tensile wires. These stress results enable fatigue analysis to be performed with a greater sophistication than has previously been possible.

HYSTERETIC RESPONSE TO BENDING

An unbonded flexible consists of a number of concentric layers, with a number of layers of helically wound tensile armour wires, wound in opposite directions, embedded between the layers; see Fig. 1. When the pipe is bent each layer generates a contribution to the total bending moment. The non-tensile layers generally bend with the pipe and generate bending moment contributions that are not hysteretic and are fairly easy to model. The behaviour of the tensile wires is more complex, since pipe bending generates tension in the tensile wires that tries to slip the wires within the structure.

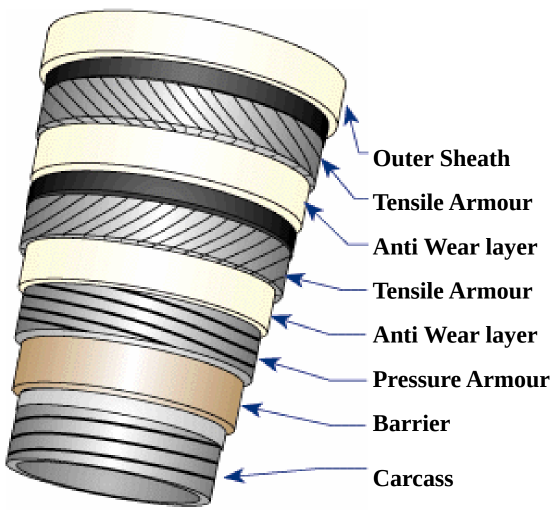

Figure 1: Flexible Pipe Construction

Consider an unbonded flexible pipe that is initially straight and unstressed and is then progressively bent. The initial straight unstressed state is represented by the point A in Fig. 2 below.

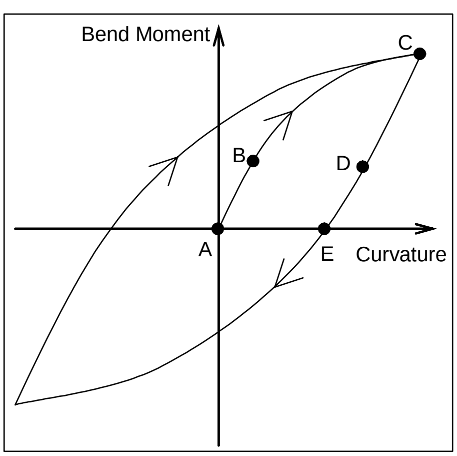

Figure 2: Bending Moment Hysteresis Loop

When the curvature is low, friction between the wires and the adjacent layers is sufficient to resist tensile wire slippage. As a result the axial stress σ_a in the tensile wires typically shows a sinusoidal profile along its arc length, as a result of the helical path of the wires. By considering the equilibrium of the wire element, as illustrated in Fig. 3, it can be seen that the gradient of the axial stress curve gives a net axial tensile force per unit length on the wire element, given by $(d\sigma_a/dL).A$. Here A is the wire cross-sectional area and L is arc length along the wire.

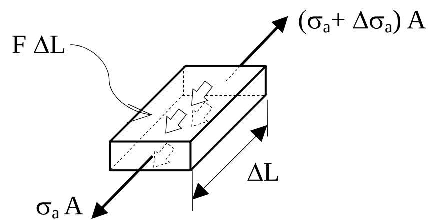

Figure 3: Axial Equilibrium of a Tensile Wire Element

This net tensile force on the element is trying to slip the wire axially, but is opposed by a distributed friction force F.ΔL, where F is the friction force per unit length on the internal and external surfaces of the wire.

While the curvature is low, friction is sufficient to maintain this equilibrium and the tensions in the armour wires, and the resulting bending moment contributions, grow linearly with curvature. So at this stage the total bending moment is a simple linear function of curvature, as represented by the line AB in Fig. 2.

But as the curvature is increased a point is reached (before the curvature limit of the pipe) where the available friction in the axial direction of the wire is insufficient to prevent parts of the wires slipping axially along their helical position within the structure. This slippage reduces any further tension increase at the extrados and compression increase at the intrados of the bent pipe, and so slippage reduces further increases in the bending moment. In other words, the slope of the moment-curvature curve actually reduces. This effect grows as more of the tensile wire slips, so the effective bending stiffness, i.e. the slope of the moment-curvature curve, reduces. This is represented in Fig. 2 by the curve from B to C, where C represents the manufacturer's curvature limit for the flexible.

If the applied curvature is now reversed then the tensile wires will not immediately slip back, since friction is now acting to hold them in their displaced position. So the wire tensions and resulting bending moment again change linearly with curvature, represented by the straight section from C to D in Fig. 2. But as the curvature reduces further a point is reached where friction can no longer hold the wires in their displaced positions, so the wires slip back and the effective stiffness reduces again. This is the curve D to E in Fig. 2. The pipe has now returned to zero bending moment but with a non-zero curvature. Further changes in curvature continue the process, giving the classical hysteresis loop graph.

ORCINA BENDING HYSTERESIS MODEL

The basic non-linear moment-curvature response of a flexible riser, represented by the curve A to C in Fig. 2, can be measured experimentally by progressively bending a sample of pipe up to its curvature limit. But in practice the curvature cycles will vary in amplitude and be below the curvature limit, so the moment-curvature curve will follow hysteresis loops of various smaller amplitudes, as illustrated in Fig. 4.

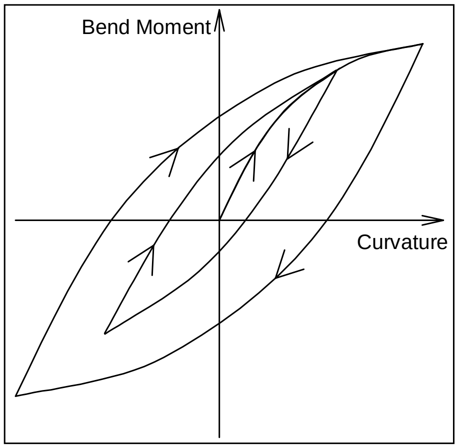

Figure 4: Response to changes in curvature amplitude

Also, the direction of curvature might change as well as the magnitude, due to the combination of wave action and roll, pitch and yaw of a top end vessel. The resulting 3D bending hysteresis would require a 3D graph to represent it.

It is clearly not possible to determine the pipe response experimentally for all possible time histories of curvature magnitude and direction. Therefore the Orcina bending hysteresis model uses the basic non-linear curve A to C and uses it to calculate the bending moment history that results from any given curvature history. It does this using a natural mathematical vector model that models hysteresis in 3D, which will now be described.

The curve A to C is specified to the model as a series of n+1 points (x_i, y_i), i=0..n, where y_i is the moment magnitude corresponding to curvature magnitude x_i, and the first point (x_0, y_0) must be (0,0). In between the i^{th} and $(i+1)^{th}$ points the curve is taken to be linear with effective bend stiffness (i.e. slope of that part of the curve) given by

$$k_i = (y_i - y_{i-1}) / (x_i - x_{i-1}) \qquad (i=1..n)$$

Beyond the last point the curve is extrapolated linearly, i.e. with effective bend stiffness $k_{n+1}=k_n$. The following Fig. 5 illustrates this for a small number of points; the user can of course specify more points to more accurately represent the curve.

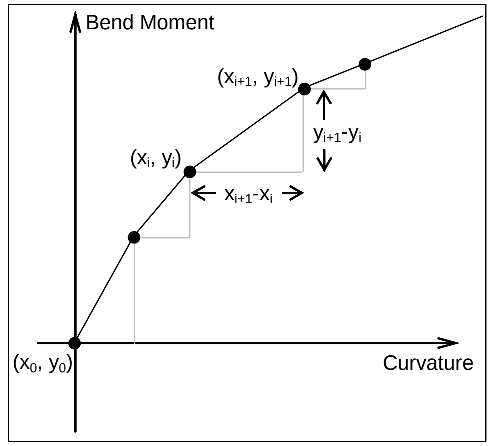

Figure 5: Data for Orcina Model

The model calculates the bending moment vector corresponding to any given curvature vector $\underline{C}$ (underscored symbols represent vectors) by representing it as a sum of n+1 curvature increments

$$\underline{C} = \underline{\delta C}_1 + \underline{\delta C}_2 + .. + \underline{\delta C}_{n+1}$$

The i^{th} curvature increment $\underline{\delta C}_i$ is then taken to generate a corresponding bending moment increment $\underline{\delta M}_i = k_i\, \underline{\delta C}_i$ that is calculated using the effective stiffness k_i of the i^{th} linear part of the curve specified in the data. So the total bending moment vector is

$$\underline{M} = k_1\, \underline{\delta C}_1 + k_2\, \underline{\delta C}_2 + .. + k_{n+1}\, \underline{\delta C}_{n+1}$$

The correct hysteretic behaviour of the model comes from the way the total curvature $\underline{C}$ is allocated into curvature increments $\underline{\delta C}_i$.

Initially the total curvature vector $\underline{C}$ and the curvature increments are all taken to be zero, corresponding to an initially straight unstressed state.

Then at any given time t in the simulation, the curvature increment vectors $\underline{\delta C}_i$ are calculated from their values at the previous time step t-δt using the change in total curvature $\underline{\delta C} = \underline{C}(t) - \underline{C}(t-\delta t)$ that has occurred since that previous time step. This is done by allocating as much as possible of $\underline{\delta C}$ to the first curvature increment $\underline{\delta C}_1$, subject to the rule

$$|\underline{\delta C}_1| \le x_1 - x_0$$

i.e. that the vector magnitude of curvature increment must not exceed the curvature magnitude span x_1-x_0 of the corresponding interval on the moment-curvature data curve.

Having allocated as much as possible to the first curvature increment, any remaining curvature change since the last time step is allocated to the next curvature increment $\underline{\delta C}_2$, subject to the limit that $|\underline{\delta C}_2| \le x_2 - x_1$, then any left to the next curvature increment, and so on. The last curvature increment $\underline{\delta C}_{n+1}$ is not limited, corresponding to the linear extrapolation of the user's data beyond the last data point (x_n, y_n), and so $\underline{\delta C}_{n+1}$ receives any curvature change that is left after as much as possible has been allocated to the earlier increments.

In other words this hysteresis model treats the curvature and moment vectors as being made up of a series of curvature vector increments and corresponding moment vector increments. Curvature change is allocated on a ‘change lowest increment possible’ basis, subject to each increment never exceeding in magnitude the corresponding curvature span in the user’s data, and the moment increments are calculated from the curvature increments using the bend stiffness appropriate to that interval in the pipe’s data table. This is a ‘first in first out’ basis, as opposed to the ‘last in first out’ basis that a non-hysteretic model would use, and its effect is that the bending moment follows the hysteresis curve wanted.

This model is a mathematically natural way of extending scalar non-linear data to vector values. It has the desired hysteretic behaviour, and the results presented later in this paper show that it gives a good representation of the bending moment response due to curvature, both when the curvature is all in a single plane and also when the plane of curvature varies.

To see how this model gives hysteretic behaviour, consider a pipe being bent in a single plane. At first the curvature is small and all of it can be accommodated in the first curvature increment $\underline{\delta C}_1$, without exceeding that increment’s magnitude limit (x_1-x_0). So the only non-zero curvature increment is $\underline{\delta C}_1$ and the overall effective bend stiffness is k_1. When the curvature magnitude reaches the first increment’s limit (x_1-x_0) that first increment can no longer be increased, so further curvature increase is allocated to the second increment $\underline{\delta C}_2$, which uses the smaller stiffness k_2, and so the overall effective bend stiffness reduces.

If the curvature is now reduced then initially the first increment can accommodate the curvature changes, since they reduce its magnitude, so initially the curvature reduction happens with the highest effective stiffness k_1. But further curvature reductions leads to $\underline{\delta C}_1$ reversing and increasing in magnitude again, and then reaching its magnitude limit but in the opposite direction, after which curvature reduction now has to be allocated to $\underline{\delta C}_2$, using a lower effective bend stiffness, and so on. The model also gives bending hysteresis for curvature in 3D, since the equations of the model are vector equations.

WELLSTREAM DETAILED PIPE MODEL

As an alternative to the simplified approach used in the Orcina model, as part of an ongoing R&D program Wellstream have developed a more sophisticated analytical model to quantify the relationship between the pipe 3D bending hysteresis and the local tensile wire behaviour. The implementation of the model enables the direct prediction of the pipe global bending responses, governed by the bending hysteresis characteristics, and also the prediction of the corresponding stress and strain in the tensile wires. This model has been implemented as an external software module for use with OrcaFlex.

Under normal operation conditions, where the pipe is subjected to both the combined internal/external pressures and tension loads, there is no separation between the tensile layers and their adjacent layers. It is therefore reasonable to assume that the wire is confined to stay on the adjacent cylindrical surface. The possible slipping of the wire over this surface can be divided into two components, namely slipping in the wire’s axial direction and slipping in the wire’s bi-normal direction (the normal to the wire's axial direction, in the surface plane of the adjacent cylinder); see Tan et al [2].

The slipping of the tensile wire in its axial direction is believed to be the primary mechanism leading the bending hysteresis behaviour, since such slipping limits the increase of the wire axial stress during bending, which directly governs the pipe bending stiffness. Slipping in the bi-normal direction has much less direct influence over the pipe bending stiffness than the slipping in the axial direction.

At each time step the Wellstream model tracks and updates the tensile wire slippage region as changes occur in the pipe curvature amplitude and direction, and then determines the corresponding axial stress for each tensile wire at each pipe cross-section. The model finally calculates the corresponding bending moment contribution from each tensile layer, with respect to the two principal bending axes of the pipe.

MODEL IMPLEMENTATION USING ORCAFLEX

The Wellstream model is implemented in a separate external function software module for use with OrcaFlex, and the interaction between the Wellstream model and OrcaFlex is illustrated in the following Fig. 6.

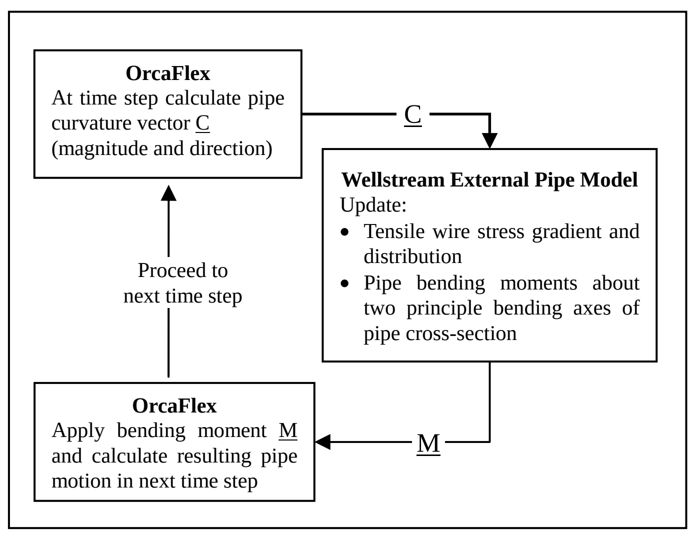

Figure 6: Interaction between OrcaFlex and Wellstream External 3D bending Hysteresis Model

Before the dynamic simulation commences, OrcaFlex calculates a riser static configuration that is taken as the initial state for the dynamic simulation. At this starting point the stress and the stress gradient due to internal friction are set to zero for each pipe element, i.e. zero friction moment.

At each subsequent time step, and for each element in the OrcaFlex model of the pipe, OrcaFlex calculates the pipe curvature vector and passes it to the Wellstream software. The Wellstream software then calculates the detailed tensile wire stress gradient and profile, based on the current and past history of curvature at each element. It then calculates the corresponding reacting bending moments, with respective to the two principal bending axes defined at the pipe cross-section, and passes these bending moments back to OrcaFlex. OrcaFlex then applies these bending moments and calculates the motion of the pipe in the next time step, allowing for environment loadings, vessel motion, etc.

OrcaFlex records the time history of the pipe motion, curvature, bending moment, effective tension, etc. The detailed tensile wire stress time histories for each wire are not recorded, but are reconstructed at the OrcaFlex post-processing stage for use in a regular or rainflow fatigue analysis in order to calculate the wire fatigue damage.

FULL SCALE PIPE BENDING TEST AND MODEL CALIBRATION

A full scale 4-inch pipe bending test was performed by SINTEF [3]. The relationship between the pipe bending curvature and moment was measured at three different internal pressure levels (7, 100 and 200 bar). Figure 7 shows the test set up and the position of the extensometers for measuring the strain. Figure 8 shows the measured bending hysteresis loops measured at different pressure levels.

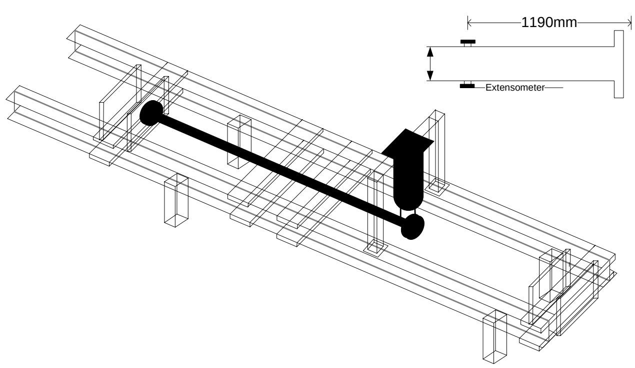

Figure 7: Test Setup – 4 inch ID pipe

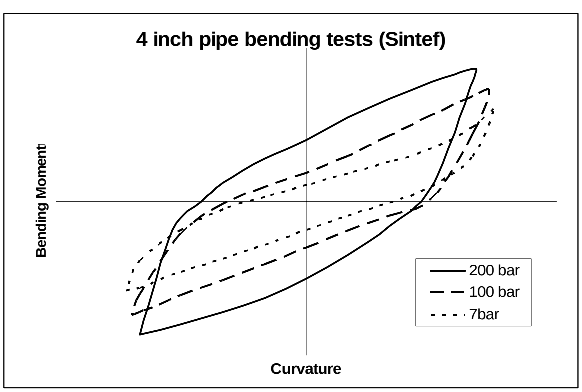

Figure 8: Typical bending hysteresis loops measured over 4 inch pipe test (normalized data)

The test results were used to calibrate the analytical 3D bending hysteresis model, in terms of the effect of the internal friction under the applied loading conditions. The internal friction governs the tensile wire slip-stick condition, and the level of the adjacent polymer layer shear deformation due to wire axial movement and the tendency of such movement.

The same loading conditions were then simulated. Figures 9, 10 and 11 show the comparisons between the model simulation and the test results, for the 3 different internal pressures.

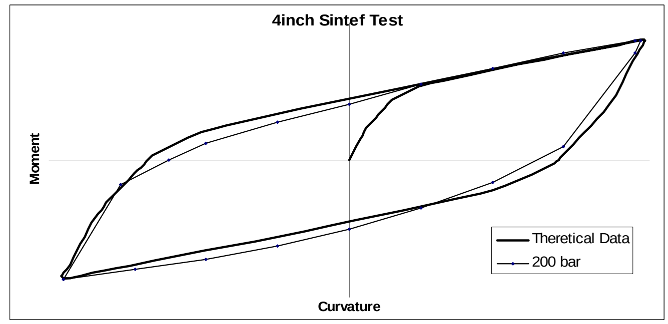

Figure 9: Comparison of the model prediction and test results at 200 bar internal pressure

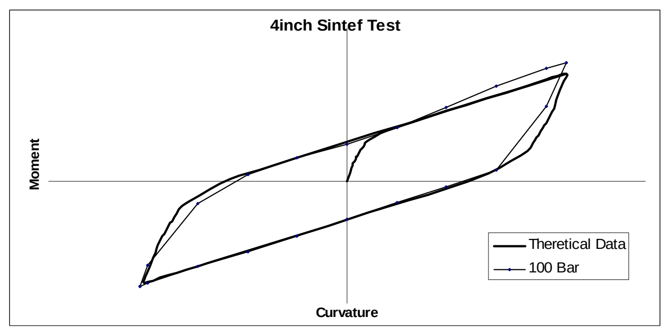

Figure 10: Comparison of the model prediction and test results at 100 bar internal pressure

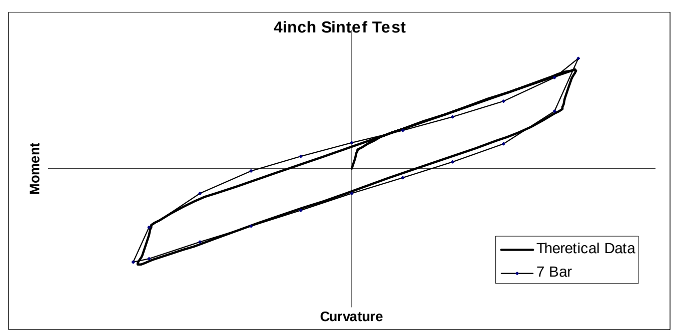

Figure 11: Comparison of the model prediction and test results at 7 bar internal pressure

DYNAMIC RESPONSES CASE STUDY

A simple case of a FPSO with a free hanging catenary riser configuration was used to study the performance of the Wellstream and Orcina models, and to compare the results of the two models. The riser configuration is illustrated in Fig. 12. The riser is 1628m long, made of the same type of 4 inch pipe as used in Sintef full scale bending test, with an internal pressure of 200 bar. The water depth was set to 1000m and the FPSO was assumed to have a turret.

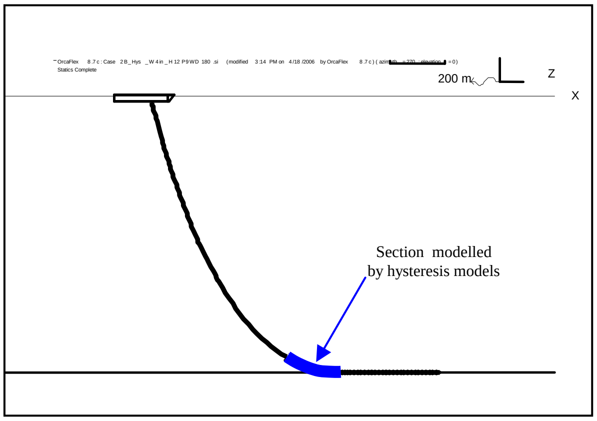

Figure 12: Study case riser configuration

The Wellstream and Orcina models were used to model the bending of a section of about 200m around the touchdown zone (TDZ), modelled using 40 elements. The curvature in the rest of the pipe was assumed to not be sufficient for the effects of bending hysteresis to be significant.

Both regular and irregular time domain dynamic simulations were performed, to test the performance of the Wellstream bending hysteresis software model with OrcaFlex. The Wellstream software works very smoothly with OrcaFlex, in both regular and irregular wave simulations, but it does slow down the simulation by a factor of 2 to 4 (dependent on the time step used). However, no numerical problem, such as divergences or stall, has been encountered even at larger time steps.

The simulations were also performed using the Orcina bending hysteresis model, using moment-curvature data predicted by the Wellstream model as input. The Orcina model does not slow down the simulation significantly.

For illustration, one of the irregular simulation cases is presented. In this study, a 3 hour irregular wave train was generated based on a JONSWAP Spectrum of peak period 9 seconds and significant wave height 3 metres. Then a short section of 100 seconds was selected that contained the maximum wave height of the wave train. This section of the wave train was then simulated, with the waves being applied in the plane of the riser in the 'far' direction, i.e. away from the anchor. No current was considered in this study. As a result, the riser shows a dominant in-plane dynamic response.

To allow comparison, the simulation was repeated with the following three different ways of modelling the bending:

- Simple linear pipe bend stiffness EI corresponding to full tensile slipping
- Nonlinear hysteretic pipe bending using the external Wellstream software model
- Nonlinear hysteretic pipe bending using the Orcina model that is built into OrcaFlex. The table of bending moment as a function of curvature that is required by this model was generated using the Wellstream model calibrated at 200 bar internal pressure.

Figure 13 shows the three trajectories of the pipe bending moment and curvature variation at TDZ during the simulation. As anticipated, both the Wellstream and Orcina models do produce typical hysteresis loop behaviour, whereas a clear linear relationship was produced by the linear model. It is also noted that the Orcina model results closely match the Wellstream model results; this is reasonable given that the Orcina model was using data based on the Wellstream model.

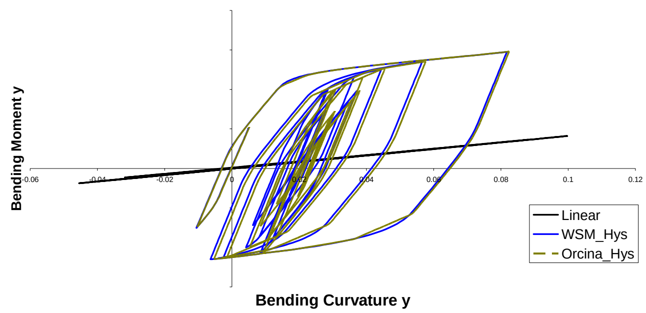

Figure 13: Comparison of moment-curvature trajectory at TDZ

The corresponding time history results at TDZ are given in Fig. 14. The curvature responses predicted by the Wellstream and Orcina model are very similar. As anticipated, a considerable reduction in the curvature amplitude is predicted by the two hysteresis models, compared to the linear stiffness model. This could have significant implications for subsequent fatigue analysis results.

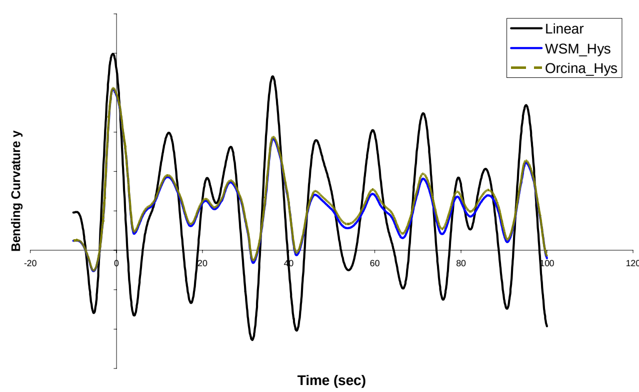

Figure 14: Comparison of curvature responses at TDZ

3D BENDING COMPARISON OF MODELS

For real applications, bending models need to handle 3D bending, i.e. where the curvature vector can vary in direction as well as magnitude. For example a catenary riser has curvature applied in the catenary plane by the catenary shape itself, and any out of plane wave excitation then excites top end roll or pitch that applies curvature in a different direction.

Experimental results were not available for cases with 3D bending, but a comparison has been made between the Wellstream and Orcina models for a simple artificial test case involving 3D bending.

The case considered was a short section of unbonded flexible pipe whose axis was in the z-direction, and to which the following history of curvature was applied. First, curvature was applied progressively about the x-axis. Then, while retaining that curvature about the x-axis, further curvature was applied about the y-axis. Then the x-axis component of curvature was gradually removed. Finally, the y- component of curvature was gradually removed, so returning the pipe to the initial zero-curvature condition.

The trajectory of this applied curvature is shown in Fig. 15. The sequence starts at the origin (0,0), i.e. zero curvature components in both the x- and y-directions, and proceeds clockwise around the trajectory. First the x-component of curvature is increased up to 0.09 rad/m (5 deg/m), then the y-component of curvature is increased to 0.09 rad/m, then the x-component of curvature is removed, and finally the y-component of curvature is removed, returning the trajectory to the origin.

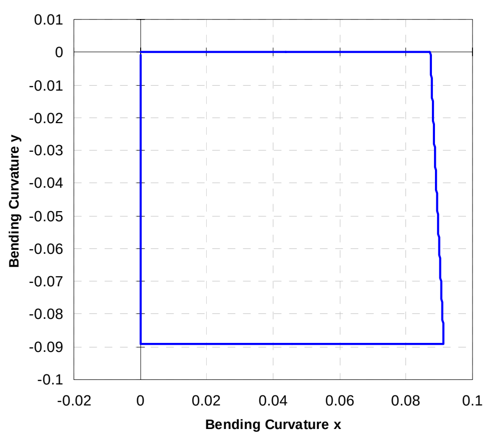

Figure 15: Applied 3D Curvature

This case was modelled using the Wellstream model, and it was also modelled using the Orcina model with curvature-moment data derived from the Wellstream model. The calculated 3D bending moment results of the two models are shown in Fig. 16.

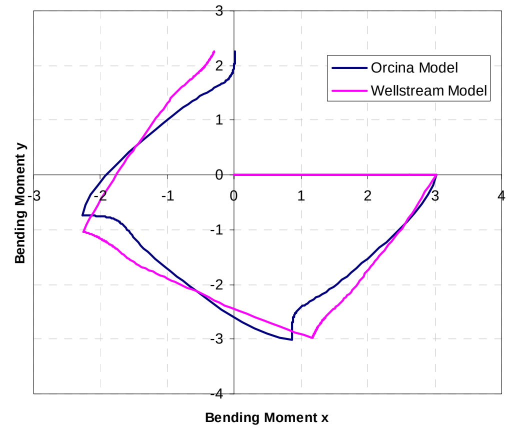

Figure 16: Calculated 3D Bending Moment

There are some differences between the bending moment trajectories that the two models predict, but the trajectories predicted by the two models, and the final bending moment values predicted, are quite similar. This gives some confidence in the way the models handle 3D bending, since the only commonality between the two models in this test is through the 2D moment-curvature data given to the Orcina model.

CONCLUSIONS AND DISCUSSION

These two models enable the important effects of bending hysteresis of flexible pipes to be modelled. The Wellstream model updates the tensile wire behaviour at each time step according to the past tensile wire behaviour and the current loadings. It can capture the full 3D bending hysteresis behaviour of the pipe during dynamic simulation, and it gives the stresses in the tensile wires for use in fatigue analysis, but it slows the simulation. The Orcina bending model does not significantly slow the simulation, and gives very similar results when given appropriate moment-curvature data, but it does not calculate the stresses in the tensile wires.

The measured bending hysteresis behavior of an unbonded flexible pipe has been reproduced well by the OrcaFlex dynamic simulation using these models. As expected, the reduction in pipe curvature response is also clearly reflected by the results obtained using the bending hysteresis models.

The Wellstream model development is an on-going process. The next step is to re-construct the tensile wire time history results at the post-process stage, and to fully utilize the existing OrcaFlex features, such as fatigue analysis using rainflow counting, to give tensile wire fatigue damage predictions. This should provide a practical and more accurate method of calculating flexible riser fatigue damage, using irregular wave time domain simulation and the rainflow method, rather than being limited to a regular wave fatigue analysis approach.

ACKNOWLEDGMENTS

The authors wish to thank to Wellstream technology management and Orcina management for continued support to this development project.

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